FREQUENCY SHIFT

KEYING:

Digital Modulation Scheme

Signal Space concept

$$P_i(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_i t)$$
$$0 \leq t \leq T$$

$$p_2(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_2 t)$$

$$0 \leq t \leq T$$

$$f_1 = \frac{k_1}{T} \qquad f_2 = \frac{k_2}{T}$$

$$T = \frac{k_1}{f_1} = \frac{k_2}{f_2}$$

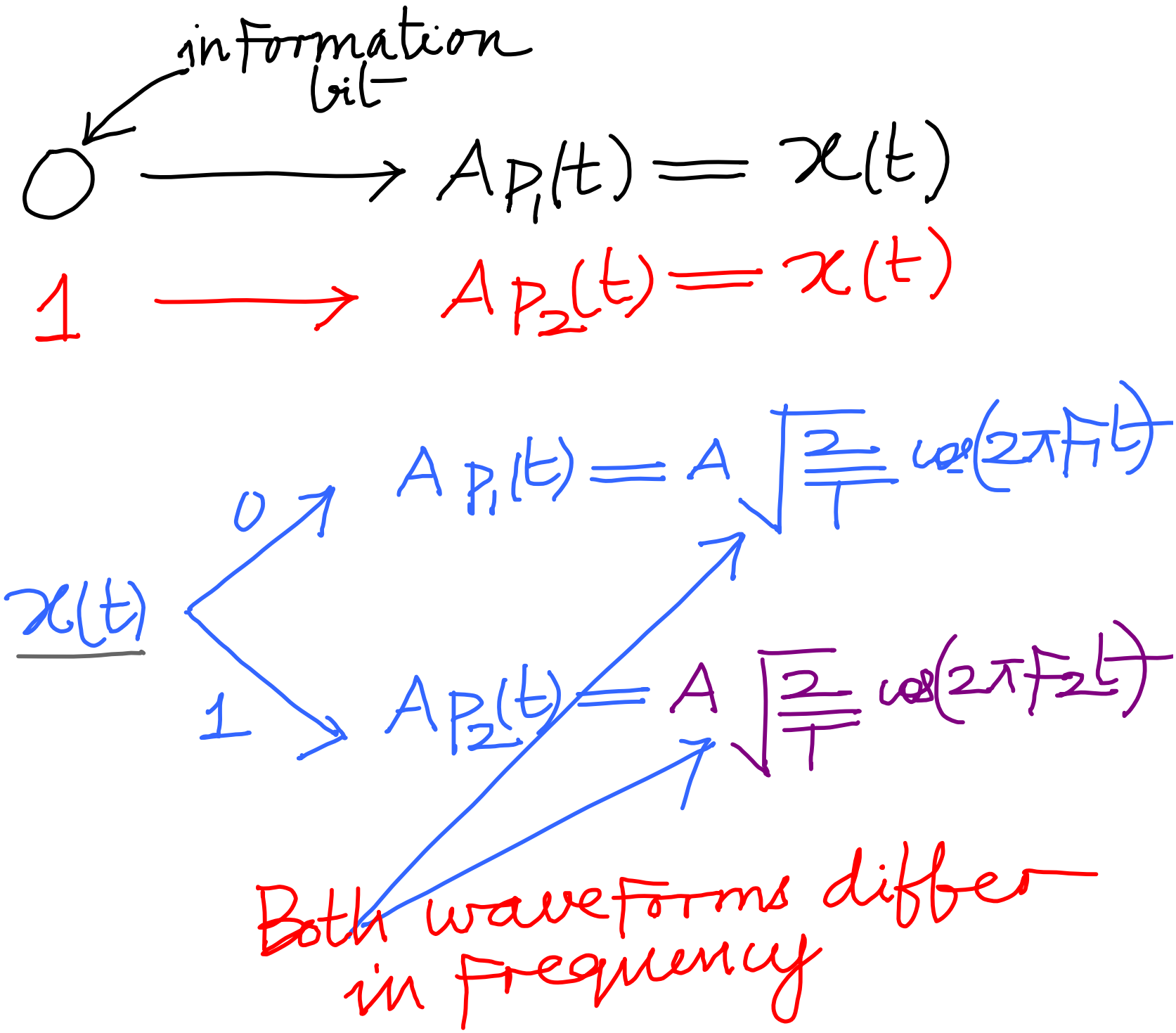
$$\mathcal{E}_p = \int_{-\infty}^{\infty} p_1^2(t)\,dt = \int_{-\infty}^{\infty} p_2^2(t)\,dt$$

$$= 1$$

Both have unit Energy

$$\int_{-\infty}^{\infty} p_1(t)\,p_2(t)\,dt = 0$$

orthogonal pulses.


information bit
$A p_1(t) = x(t)$
1
$A p_2(t) = x(t)$
$A p_1(t) = A\sqrt{\frac{2}{T}}\cos(2\pi f_1 t)$
0
$x(t)$
1
$A p_2(t) = A\sqrt{\frac{2}{T}}\cos(2\pi f_2 t)$
Both waveforms differ in frequency

one is obtained from other by shifting in frequency

Hence, this modulation scheme is termed as Frequency Shift keying (FSK)

Assuming 0, 1 occur with probability $= \frac{1}{2}$ each,

$$\text{Average energy} = \frac{1}{2} A^2 \mathcal{E}_p + \frac{1}{2} A^2 \mathcal{E}_p .$$

$$= A^2 \mathcal{E}_p .$$

$$A^2 \underbrace{\mathcal{E}_p}_{1} = \mathcal{E}_b .$$

constant energy/bit $\mathcal{E}_b$.

$$\Rightarrow A^2 = \mathcal{E}_b .$$

$$\Rightarrow \boxed{A = \sqrt{\mathcal{E}_b} .}$$

$x(t)$

0 : $\sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_1 t)$

1 : $\sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_2 t)$

Receiver

$$y(t) = x(t) + n(t)$$

Additive white Gaussian Noise

$\frac{N_0}{2}\delta(\tau) = R_{nn}(\tau)$.

$$0 \longrightarrow y(t) = A p_1(t) + n(t)$$

$$1 \longrightarrow y(t) = A p_2(t) + n(t)$$

Two pulses $p_1(t)$, $p_2(t)$.

Matched Filter.

$h(t) = p_1(T-\tau)$?

$= p_2(T-\tau)$?

Which pulse should be chosen for matched Filtering ?

$$\tilde{y}(t) = y(t) - A\left(\frac{p_1(t) + p_2(t)}{2}\right)$$

Constant signal.

For 0, we have,

$$y(t) = A p_1(t) + n(t)$$

$$\tilde{y}(t) = y(t) - A\left(\frac{p_1(t) + p_2(t)}{2}\right)$$

$$= A\left(\frac{p_1(t) - p_2(t)}{2}\right) + n(t)$$

$$\underbrace{\qquad\qquad}_{\tilde{p}(t)}$$

$$= A\tilde{p}(t) + n(t).$$

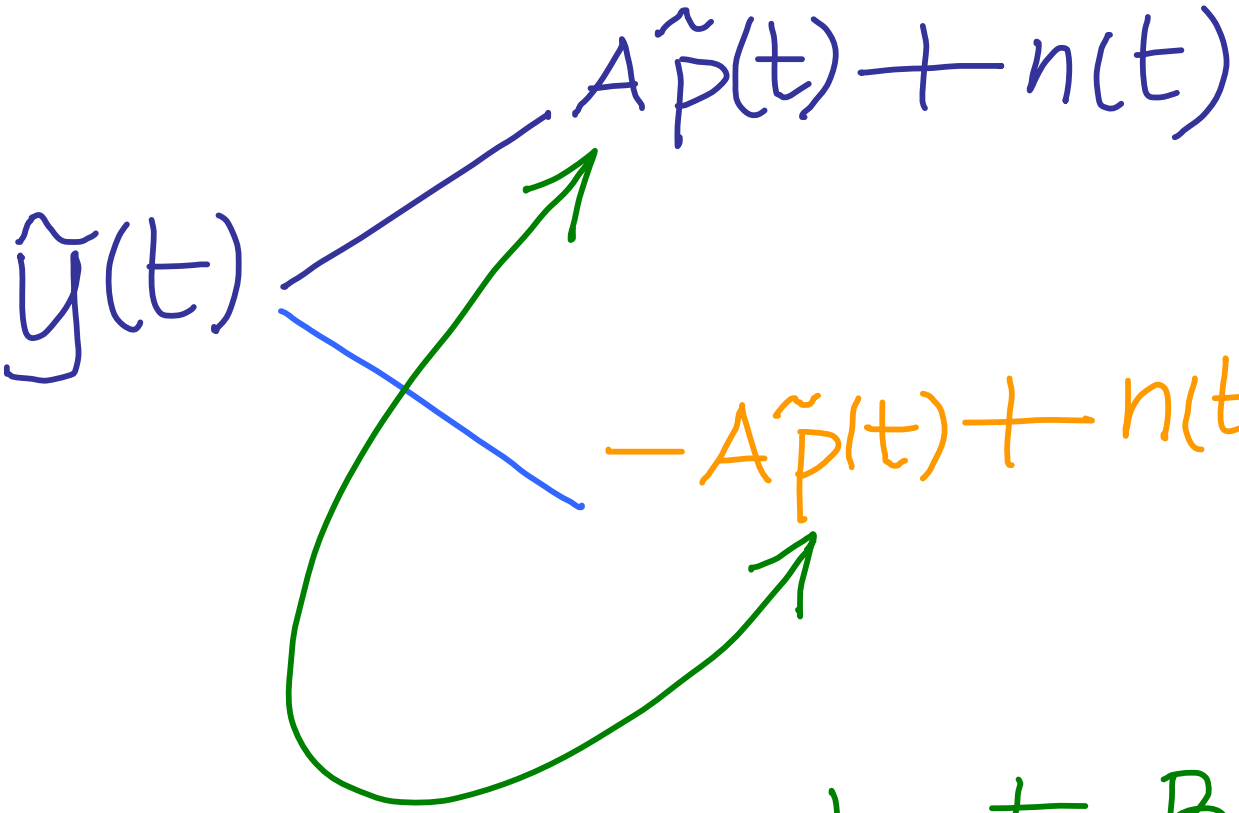
For information bit = 1

$$y(t) = A p_2(t) + n(t)$$

$$\hat{y}(t) = y(t) - A\left(\frac{p_1(t) + p_2(t)}{2}\right)$$

$$= A\left(\frac{p_2(t) - p_1(t)}{2}\right) + n(t)$$

$$= -A\tilde{p}(t) + n(t).$$



This is similar to BPSK and is obtained by replacing $p(t)$ by $\tilde{p}(t)$

$$\tilde{p}(t) = \frac{p_1(t) - p_2(t)}{2}$$

Therefore, the optimal matched filter is,

$$h(t) = \tilde{p}(T-\tau)$$

matched to $\tilde{p}(t)$

$= \frac{p_1(t) - p(t)}{2}$

Factor of $\frac{1}{2}$ is simply a scaling factor which does NOT affect SNR

Hence, optimal matched Filter

$$h(t) = p_1(T-\tau) - p_2(T-\tau)$$

$$= \sqrt{\frac{2}{T}}\cos(2\pi f_1(T-\tau)) - \sqrt{\frac{2}{T}}\cos(2\pi f_2(T-\tau))$$

optimal Filter at Receiver which maximizes the SNR.